International Journal of Theoretical Physics, Vol. 36, No. 11, 1997

What Do We Know and What Can We Learn About the Topology of the Universe?

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Received June 25, 1997

We review the main mathematical concepts of cosmic topology and the main observational methods to study the topology of the universe. We then show how topology can play a crucial role in the early universe.

1. INTRODUCTION

The goal of scientific cosmology is to answer a set of questions concerning the structure, origin, evolution, and fate of the universe. Within the framework of general relativity, one tries to build some plausible models for our universe. Thus, a large set of questions we would like to answer can be reset under the more abstract form: "What are the geometry and the topology of the universe?"

Einstein's equations can help us answer the first part of this question. However, since they are partial differential equations, they describe only local properties of spacetime and therefore cannot give us an answer about the global structure of our universe, i.e., its topology.

Historically, this point arose in 1917 when Einstein proposed the first cosmological solution of general relativity. The Einstein static universe has spatial sections which are 3-spheres (S^3), but de Sitter (1917) noticed that this solution can fit with another topology, the 3-dimensional projective space (P^3) constructed from S^3 by identifying antipodal points. The two solutions have the same metric, but different topologies, which reflects the choice of different boundary conditions.

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From a philosophical point of view, many arguments have been used to answer the question of the topology. On the one hand, the argument of "simplicity" or of "economy" was given to argue that the universe should be simply connected. However, this argument is so vague that it can also be used to claim the contrary. On the other hand, there are some arguments for considering compact (i.e., finite) universes: following Einstein (1955) and Wheeler (1968), we can advocate a finite universe on the basis of Mach's principle; another argument against an infinite universe was that they are "unaesthetic" (Ellis, 1975), because "anything that can happen does happen and an infinite number of times" (Blanqui, 1872; Epicurus, 1971) the only possibility for a simply connected universe to have compact spatial sections is to be closed (that is, of positive curvature). However, in a multiconnected framework, finiteness is a less stringent condition since all multiconnected universes (whatever their curvature) have compact spatial section. Hence, even if they do not directly deal with multiconnectedness, all these arguments on the finiteness of the universe certainly give good reasons to study multiconnectedness.

Other arguments to consider finite universes come from quantum cosmology (Zel'dovich and Starodinsky, 1984), since the probability of birth of the universe is argued to be inversely proportional to its action and so to its volume (in other words, "the smaller, the more probable").

Philosophy and quantum cosmology give arguments in favor of multiconnected universes and at least reasons to study cosmic topology, but none of them can give us a clear answer. Hence, one must have a more pragmatic approach and try to study the implications of a nontrivial topology for our observed universe. Ideally, this will enable us to determine the topology observationally and set bounds on the characteristic lengths of such a universe.

This way of studying a postulate is not new and has been used for another assumption of cosmology, "the cosmological principle," which evolved from the status of principle to the status of a working hypothesis (Maartens *et al.*, 1995), which has been scientifically tested by confronting inhomogeneous and anisotropic cosmologies with observations.

If topology is a relevant observational property of the universe (roughly, if at least one characteristic length of the universe is smaller than the horizon), one can try to study this property objectively. This has been initiated using the distribution of galaxies and quasars, and the cosmic microwave background (CMB).

After a brief summary of the useful properties of topology applied to cosmology (Section 2). I describe what can be learned from the distribution of galaxies and clusters (Section 3) and from the CMB (Section 4). I finish by presenting new constraints on the creation of topological defects in a multiconnected universe (Section 5).

The three first points have been developed in a review by Lachièze-Rey and Luminet (1995), where an extended bibliography can be found.

2. BASIC FEATURES OF COSMIC TOPOLOGY

I will sum up the important definitions and notations, give some theorems, and make some remarks that are useful when one applies topology to cosmology. More detailed approaches can be found in many textbooks (e.g., Nakahara, 1990).

Topology is the part of mathematics which studies continuity and tries to find out which properties remain invariant under a continuous group of transformations.

Let us consider a manifold $\mathcal{M}$ and a loop γ at $x \in \mathcal{M}$ which is a path starting and ending at the same point x.

If two loops at x, γ and γ', can be continuously deformed into one another, then we will say that they are *homotopic* ($\gamma \sim \gamma'$).

The manifold $\mathcal{M}$ will be *simply connected* if $\forall x \in \mathcal{M}$, any two loops at x are homotopic. A straightforward consequence of this is that any loop is contractible. If there is at least one loop that cannot be shrunk to a point, $\mathcal{M}$ is said to be *multiconnected* (e.g., R^n and S^n are simply connected and the n-hypertorus T^n, is multiconnected).

We can now introduce the group of loops at x, called the *first homotopy group at x*, also known as the fundamental group $\pi_1(\mathcal{M}, x)$. If $\mathcal{M}$ is arcwise connected (which is an assumption for our universe), $\pi_1(\mathcal{M}, x)$ and $\pi_1(\mathcal{M}, x')$ are isomorphic and we will denote them $\pi_1(\mathcal{M})$. This quantity is a topological invariant of $\mathcal{M}$. For instance, for the two-dimensional T^2, we can check that $\pi_1(\mathcal{M})$ is isomorphic to $Z \oplus Z$ [since the loops can wind i times around the hole and j times around the body of the torus, a given loop can be characterized by the pair (i, j) of integers].

This can be generalized to define $\pi_n(\mathcal{M})$, the n^{th} group of homotopy, the equivalence class of the "n-dimensional loops" (e.g., a "2-dimensional loop" is a surface, etc.).

A multiconnected manifold is conveniently described by its *fundamental polyhedron* $\mathcal{P}$, which is convex with a finite number of faces $\{\mathcal{F}_i\}$ identified by pairs, together with the *holonomy group* Γ, consisting of the collection of transformations γ which carry a face to its homologous face. The "γ's" are isometries without fixed points (except for the identity). For instance, for a hypertorus T^3, $\mathcal{P}$ is a cube.

$\mathcal{P}$ is transformed into its image $\gamma\mathcal{P}$ by the action of $\gamma \in \Gamma$. The set $\mathcal{U} = \{\gamma\mathcal{P}, \gamma \in \Gamma\}$ is called the *universal covering space*, and can be seen as the "unwrapping" of $\mathcal{M}$ by Γ. One has the two basic properties:

- $\mathcal{M} = \mathcal{U}/\Gamma$
- If $\mathcal{M} = \mathcal{U}$, then $\mathcal{M}$ is simply connected

In cosmology, we will identify $\mathcal{P}$ with the physical space where physical objects (galaxies, quasars, ...) lie, whereas $\mathcal{U}$ is identified with the observed universe.

One can go further and define more topological invariants to classify topological spaces. Such a classification can be found in Lachièze-Roy and Luminet (1995). Let us now stress some more points that are relevant for cosmology.

First of all, a multiconnected universe will have length scales associated with its fundamental polyhedron; only two of them will be considered, α (the smallest length of the polyhedron) and β (the maximum length inscribed in the polyhedron). For instance, in a hypertorus, α is the length of the smallest edge of $\mathcal{P}$ (a parallelepiped), and β is the length of the "longest" diagonal of $\mathcal{P}$.

We will assume that the universe is isotropic and homogeneous on the large scale and thus that it can be described by Friedmann–Robertson–Walker metric. If the universe is simply connected, then the spatial sections are finite or infinite according to the sign of the spatial curvature index K. Euclidean ($K = 0$, also called "flat") and hyperbolic ($K = -1$, also called "open") models have an infinite volume, whereas elliptical ($K = +1$, also called "closed") models have a finite volume. However, when the topology is nontrivial, all of these three geometries can have compact sections. It is worth stressing that the words "open," "flat," and "closed" apply to the local geometry and that an open universe can have compact (and thus finite) spatial sections. With such cosmological models, the universal covering space can be identified either with S^3 if $K = +1$, or R^3 if $K = 0$, or H^3 if $K = -1$. Let us also note that it is impossible to distinguish a strictly periodic and simply connected universe from a multiconnected universe. But all the properties of the universe have to be strictly periodic, which is most unlikely.

One also has constraints on the universe coming from time orientability and causality conditions (Hawking and Ellis, 1973). We will assume that the spacetime is *globally hyperbolic* (it implies that it is stably causal and thus time-orientable), which means that it has *Cauchy surfaces*, i.e., that the knowledge on a hypersurface can enable us to determine the information in the whole spacetime.

Under this assumption it can be shown that the study of the topology of spacetime reduces to the study of the topology of the constant-time hypersurfaces (Sokolov, 1975).

We can also wonder if we should also ask for the space to be orientable, although it is less stringent than the requirement of time orientability. Some arguments have been given, based on the *CPT* theorem (see Lachièze-Roy and Luminet, 1995, and references therein). We will not make such an assumption here.

The last point we want to discuss here deals with observations. An observer making an observation records a set of values (r_{obs}, θ, ϕ) of coordinates.

All information comes to the observer through light rays along null geodesics of the spacetime. In a simply connected universe, there is one and only one null geodesic relating a given object and the observer (if we neglect local gravitational lensing) and there is a bijective correspondence between observed coordinates and real positions of the objects from which we can deduce general laws (e.g., the redshift increases with the distance to the object, etc.). In a multiconnected universe, this correspondence may not hold, and many null geodesics starting from a given object can reach the observer (since they can make many "turns around" the universe). A given object will then be associated with many images; the nearest image is called "real" and the others are called "ghosts." A way out of this problem is to work in the universal covering space $\mathcal{U}$ where all ghosts are associated with an image of the physical object by an element of the holonomy group Γ. Then there will be only one null geodesic between two points.

Working in the universal covering space also has another advantage since the horizon in $\mathcal{U}$ has the same definition as in a simply connected universe. Hereafter (and particularly in Section 4) the "horizon" will refer to the horizon defined in $\mathcal{U}$.

3. THE UNIVERSE AS A CRYSTAL

The first attempts to investigate the topology of the universe were based on the last property we have described, i.e., the existence of multiple images of the same object.

However, there are some effects that make the observation of ghosts not so simple. The first one lies in the fact that one has to take evolution effects into account since different images correspond to different epochs in the object's life. Of course, because the universe is not strictly homogeneous, the null geodesics are deformed by gravitational lensing and the deformation, amplification, and multiplication of images should, in principle, be considered; however, in general, these effects are weak enough to be neglected. Finally, proper velocities must also be taken into account. During the time t_u needed to travel around the universe, the real object has moved by $d \sim V_{proper} t_u$. Hence, the position of the next ghost is shifted by the corresponding angle. Observationally this fixes a limit on the needed spatial resolution (typically a few arcminutes).

The first idea is to use specific objects. Many authors (e.g., Sokolov and Shvartsman, 1974) tried to recognize our own galaxy and gave a minimum value of the identification length $\alpha > 15\ h^{-1}$ Mpc. Gott (1980) found, using the Coma cluster, that $\alpha > 60\ h^{-1}$ Mpc.

Let us point that this family of tests, which involve discrete sources, must consider populations of objects extending deep enough in space to check large dimensions. Quasars seem to be good candidates, since they occupy a large volume of the universe and have a strong optical luminosity, but their estimated lifetime (a typical lifetime is $\approx 10^8$–10^9 years) prevents using them to study scales bigger than 30–300 Mpc.

The clusters and superclusters seem in fact more useful. Most of the bounds on the topology come from these sources. For instance, the number N of ghost images for a given real object in a cubic hypertorus of characteristic length L in h^{-1} Mpc is given in Table I (Lehoucq *et al.*, 1996).

Different methods, such as the search for periodic distributions of objects in redshift or in distance (Fang, 1990), have been proposed, since it is tempting to use topology to explain the apparent periodicity in a catalog of galaxies (Broadhurst *et al.*, 1990).

Most of the various methods developed to test the cosmic topology have at least one of the following three drawbacks:

- They are designed for a given topology and then can set bounds or exclude only one given configuration (most of the time the hypertorus). None of them can extract a signature of the topology from the observations.
- They use very strong hypotheses concerning the cosmological model and the cosmic objects (population, evolution, . . .).
- As we stressed before, they suffer from many observational problems, including evolution effects and absorption by the interstellar medium, which make it hard to identify objects (particularly when one wants to recognize the ghost image of a given object).

Recently, a promising approach has been proposed by Lehoucq *et al.* (1996). A statistical method was developed based on the construction of pair

Table I. Number N of Ghost Images for a Given Real Object in a Cubic Hypertorus of Characteristic Length L (Lehoucq *et al.*, 1996)

L (h^{-1} Mpc)	$N(z < 1000)$	$N(z < 4)$	$N(z < 1)$
500	7000	1200	180
1500	279	45	7
2500	60	10	1.5

separation between cosmic sources and relies on the fact that each image of a given object is linked to the other images by the holonomies of space. Among all the pairs of all images, the ones formed by two ghosts of the same object reflect an isometry of the space (those pairs are called "gg-pairs").

When one draws a histogram of the pair separation, these "gg-pairs" will emerge from ordinary pairs as a spike. The location and the relative height of the spikes are a signature of the topology. The test will then consist in recognizing the presence of these pairs in a 3D catalog of observed cosmic sources.

The authors applied this method to a cluster catalog containing 901 clusters up to a redshift of $z \sim 0.35$. No significant spikes were observed, which set the bound $\alpha > 650\ h^{-1}$ Mpc.

One of the key points for implementing all these methods is to possess a 3D catalog of cosmic objects. Unfortunately, existing catalogs have a very good angular resolution but a low depth in redshift. Thus it can be hoped that future observational programs devoted to redshift surveys will enable us to use such methods with more accuracy within the next decades.

4. STUDYING THE COSMIC TOPOLOGY WITH THE CMB

All the photons propagating from the last-scattering surface have been emitted roughly at the same epoch (at a redshift $z \approx 1000$). Thus one obvious advantage of the CMB for the study of topology is that one does not have to take into account evolution effects.

The fluctuations in the observed temperature of the CMB are interpreted as the effects of inhomogeneities on the last-scattering surface and on the path between the last diffusion and us. The temperature contrast $\Delta \equiv \delta T/T$ is related to the gravitational potential via the Sachs–Wolfe effect (Sachs and Wolfe, 1967)

$$\Delta(\mathbf{n}) = \left[\frac{1}{4} D_r - v_j n^j + \Psi - \Phi\right]_{\rm em}^{\rm obs} - \int_{\rm em}^{\rm obs} (\dot{\Psi} - \dot{\Phi})\, d\eta$$

where D_r is the gauge-invariant density perturbation of the radiation fluid, v^j the gauge-invariant velocity of the baryon fluid, n the direction of observation, and Ψ and Φ the two gauge-invariant scalar potentials. The first term represents the intrinsic fluctuations on the surface of last scattering, the second term is a Doppler shift, the third and last terms are gravitational redshift contributions (including the difference of potential between the observer and the emitter and the time dependance of the gravitational potential). For a review on the perturbations of the microwave background see, e.g., Durrer (1994, 1997).

It is convenient to decompose Δ into spherical harmonics

$$\Delta(\mathbf{n}) = \sum_{l=2}^{l=\infty} \sum_{m=-l}^{m=+l} a_{lm} Y_{lm} (\theta, \phi)$$

with

$$a_{lm} = \int \Delta(\mathbf{n}) Y_{lm} \, d\Omega$$

Since we have a global isotropy of the CMB, it is sufficient to consider the terms

$$a_l^2 = \frac{1}{2l + 1} \sum_{m=-l}^{m=+l} |a_{lm}|^2$$

In a simply connected universe, one can show that for the so-called Harrison–Zel'dovich power spectrum for initial perturbations one has (Stevens *et al.*, 1993)

$$a_l^2 \propto \frac{1}{l(l + 1)}$$

In a multiconnected universe, the situation is different. Indeed, the spectrum of "allowed" wavelengths becomes discrete, for instance, in the case of a hypertorus,

$$\mathbf{k} = 2\pi \left(\frac{n_1}{L_1}, \frac{n_2}{L_2}, \frac{n_3}{L_3} \right)$$

where the L_i are the three scales associated with $\mathcal{P}$. One can see that there will be a maximum allowed wavelength $\lambda_{max} \propto \beta$. This will modify the spectrum of the temperature anisotropies in two ways:

- The ratio between temperature fluctuations at large and small angular scales is decreased because there is no source at large scales (there is a "cutoff" due to the size of the fundamental polyhedron). Moreover, fluctuations at large scales are created as a "queue effect" of spatial fluctuations at smaller scales.
- The dependence on θ at large scales is also modified.

Both effects have been studied by Stevens *et al.* (1993), Starobinsky (1993), and recently by de Oliveira-Costa *et al.* (1995). They all claim that the four-year COBE data exclude a multiconnected universe with $\beta < R_{\text{horizon}}$.

Other tests were proposed, including one by Starobinsky (1993) using l_m such that $\sqrt{2l + 1}a_l$ is maximum. The COBE data give the bound

$l_m < 7$. When applied to a flat universe with a hypertorus topology, Starodinsky found that $\alpha \sim 9000\ h^{-1}$ Mpc. This result is also valid if the universe is compact only in some directions.

Cornish *et al.* (1996) have proposed a new method to study the topology with the CMB. It is based on the following remark. The observed last-scattering surface is a 2-sphere. If the fundamental polyhedron is smaller than this sphere, then the sphere will intersect with the polyhedron on circles. Therefore, the CMB fluctuations will be correlated on circles of the same radii on different points of the sky. The number of these circles and their mutual location is a signature of the topology.

It therefore looks as if these results rule out nontrivial topologies on sub-horizon scales. However, one must stress that they rely on a strong assumption, which is that the universe is flat [the only study considering an open universe was done by Fagundes (1993)].

Now, it is not obvious that what happens in the flat case is generic to all universes. In a flat or closed multiconnected universe, there is a maximum allowed wavelength, but let us stress that these studies give a bound on β which can be from 3 to 10 times bigger than the scales of the polyhedron. Therefore what is shown is that $L \approx 300$–$1500\ h^{-1}$ Mpc.

In an open universe the situation is completely different, since there is no cutoff for the allowed wavelengths [and the number of modes grows exponentially with the wavelength; see Balazs and Voros (1986)]. This invalidates the methods used in the flat case.

In conclusion, what has really been shown is only that the topology of the hypertorus, if the universe is flat, is ruled out.

5. TOPOLOGICAL DEFECTS AND THE NO-DEFECT CONJECTURE

All the previous approaches try to characterize the topology of the universe through the observation of objects or the cosmic microwave background.

Another way was proposed recently by Uzan and Peter (1996), who studied the influence of multiconnectedness on the physical processes in the early universe.

The production of topological defects during the early-universe phase transitions where a symmetry group G is broken in a smaller group H depends only on the topology of the vacuum manifold G/H (not to be confused with the topology of the universe!). If $\pi_0(G/H) \neq \{Id\}$, then domain walls must form, whereas strings and monopoles form respectively if $\pi_1(G/H) \neq \{Id\}$ and $\pi_2(G/H) \neq \{Id\}$, where $\pi_n(G/H)$ refers to the nth homotopy group.

When one considers a multiconnected universe, the situation is different since the total winding number of all closed paths lying in the fundamental polyhedron must vanish. This observation has a lot of consequences in cosmology [see Uzan and Peter (1996) for all the details concerning the mathematical proof of this].

The physical discussion implies two quantities

$$\Upsilon \equiv \frac{\xi}{L} = \frac{\ell_p}{L_p} \qquad \text{and} \qquad \Xi_o \equiv \frac{L_0}{H_o^{-1}}$$

where ℓ_p is the Planck length, L_p the cell size (L) at the Planck time, ξ the correlation length of the defect, and H_o the Hubble constant today.

Let us note that in the case where topology comes from the quantum to classical gravity transition, one may expect $L_p \sim \ell_p$ and thus $\Upsilon \sim 1$. We let this parameter free to be more general (indeed, there is another possible length scale in the problem, the inverse square root of the cosmological constant, if it does not vanish, $l_\Lambda = \Lambda^{-1/2}$).

I summarize our conclusions concerning the existence of defects if we assume that the universe is multiconnected.

• *Monopoles*: They can only be formed pairwise with vanishing total index, which is a quite special field configuration. It should be clear that when $\Upsilon \sim 1$ the probability to have formed a monopole is in fact much smaller than that of forming no monopole at all. The main difference between a simply connected universe and a multiconnected universe lies in the fact that the former has an infinite number of correlation volumes and the latter has only Υ such volumes, and thus that no ergodic principle can be applied (Uzan and Peter, 1996).

Therefore, if we ever observe a monopole, it will mean that the universe is simply connected or that the phase transition which gave birth to this monopole took place after reheating, or that $\Upsilon >> 1$.

• *Cosmic strings*: If they are contractible, they will decay within a time of order L/c. For uncontractible strings, the situation is slightly more involved. They have to form with a total winding number that vanishes on all path lying on $\mathcal{P}$, which is very restrictive. Moreover, since the characteristic collision time for two such strings is of order L/c, the system, being equivalent to no string, will effectively contain no string after such a time.

Therefore, the observation of a cosmic string will mean that either the universe is simply connected or $\Upsilon \not\sim 1$. In this latter case the relevant parameter is Ξ_o. The observation of a cosmic string will imply that $\Xi_o > 1$, since otherwise all strings will have decayed. Thus this will mean that the universe is simply connected up to the scale of the horizon and therefore that multiconnectedness is not a relevant property for observational cosmology.

• *Domain walls*: As for strings, either they are contractible and they will decay within a time smaller than the Hubble time or they are uncontractible. In the latter case, the mathematical proof (Uzan and Peter, 1996) states that a single wall cannot be formed and that many walls can be formed only in very special configurations. Let us, moreover, emphasize that in the standard framework of cosmology a single wall is excluded for physical reasons (Kibble, 1976, 1980). In a multiconnected universe one can only have at least two walls in a very special configuration. The topological argument and the physical argument lead us to the conclusion that domain walls cannot appear in a multiconnected universe.

The alternative therefore is simple: either a topological defect is observed unambiguously, which means that the universe is simply connected up to the horizon size; or one can show using one of the previous methods that the universe is multiconnected and we will be able to conclude that topological defects are unlikely to exist.

6. CONCLUSION

Nontrivial topologies are not excluded, even if they are constrained (we know, for instance, that the topology of the hypertorus is excluded).

Observationally, we possess two new statistical methods using, respectively, the distribution of clusters and superclusters and the cosmic microwave background. In addition, as we have seen, multiconnectedness imposes strong constraints on the formation of topological defects in the early universe.

However, some problems remain, even if a nontrivial topology is new way out for the monopole problem. In the standard cosmological framework, this problem is solved by inflation, which dilutes the monopoles, but only if it takes place after the phase transition that gave birth to the defects. In a multiconnected universe, they simply do not form; however multiconnectedness needs inflation, since the present universe will be too small otherwise.

The second problem arises from the size of the universe since we have seen that the two length scales present are the Planck scale l_p and a scale associated with the cosmological constant l_Λ. Thus, if the topology is not trivial, one has to explain the coincidence $(\alpha, \beta) \sim R_{\text{horizon}}$.

To conclude, we can maintain that we have some powerful tools and results to study the cosmic topology objectively. The "philosophical" assumption of "simple-connectedness of the universe" joins the set of testable working hypotheses, as the "cosmological principle" did.

ACKNOWLEDGMENTS

I would like to thank N. Deruelle, J.-P. Luminet, and P. Peter for reading this manuscript, and Mme Mady Smet for being our host in Peyresq.

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